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SA402 · Dynamic and Stochastic Models Fall 2022 · Uhan

Quiz 5 – 10/12/2022

Problem	Weight	Score
1	1	
2	1	
3	1	
4	1	
Total		/ 40

Instructions. You have 15 minutes to complete this quiz. You may use your plebe-issue calculator. You may not use any other materials (e.g., notes, homework, website).

Show all your work. To receive full credit, your solutions must be completely correct, sufficiently justified, and easy to follow.

For Problems 1-3, consider the following setting.

Vehicle arrive at the Simplexville Bridge toll plaza between 7 a.m. and 7 p.m. according to a nonstationary Poisson process with integrated rate function

$$\Lambda(\tau) = \begin{cases} 20\tau & \text{if } 0 \le \tau < 2 \\ 5\tau + 30 & \text{if } 2 \le \tau < 10 \\ 25\tau - 170 & \text{if } 10 \le \tau \le 12 \end{cases}$$

where τ is in hours, $\tau = 0$ corresponds to 7 a.m., and $\tau = 12$ corresponds to 7 p.m.

Problem 1. In words, briefly describe the meaning of $\Lambda(4) = 50$ in the context of this problem.

See Problem 1c from the Lesson 6 Exercises for a similar example.

Some of you wrote that $\Lambda(4) = 50$ is the number of vehicles to arrive by $\tau = 4$, or 11 a.m. This is close. Will there always be 50 vehicles arriving by $\tau = 4$?

Problem 2. What is the probability that the 45th vehicle arrives at the toll plaza at or before 11 a.m.?

To set up the probability statement correctly, consider the following question: if the 45th vehicle arrives at the toll plaza at or before 11 a.m., how many vehicles must have arrived at the toll plaza at 11 a.m.?

For hints on the computation, see Example 2d in Lesson 6 and Problem 1b from the Lesson 6 Exercises for similar examples.

Problem 3. If exactly 100 customers have arrived by 4 p.m., what is the probability that 135 or fewer customers will arrive by 6 p.m.?

See Example 2d in Lesson 6 and Problem 1b from the Lesson 6 Exercises for similar examples.

Problem 4. Vehicles arrive at the nearby Turing Tunnel toll plaza between 7 a.m and 7 p.m according to a nonstationary Poisson process with arrival rate function

$$\lambda(\tau) = \begin{cases} 9 & \text{if } 0 \le \tau < 3 \\ 3 & \text{if } 3 \le \tau < 9 \\ 7 & \text{if } 9 \le \tau \le 12 \end{cases}$$

What is the integrated rate function for this nonstationary Poisson process?

See page 1 and Example 2a in Lesson 6 for similar examples.

Exponential random variable with parameter λ:	cdf $F(a) = \begin{cases} 1 - e^{-\lambda a} & \text{if } a \ge 0 \\ 0 & \text{if } a < 0 \end{cases}$	expected value $= 1/\lambda$
Erlang random variable with parameter λ and n phases:	cdf $F(a) = \begin{cases} 1 - \sum_{j=0}^{n-1} \frac{e^{-\lambda a}(\lambda a)^j}{j!} & \text{if } a \ge 0 \\ 0 & \text{if } a < 0 \end{cases}$	expected value $= n/\lambda$
Poisson random variable with parameter λt:	pmf $p(n) = \frac{e^{-\lambda t}(\lambda t)^n}{n!}$ for $n = 0, 1, 2, \ldots$	expected value $= \lambda t$